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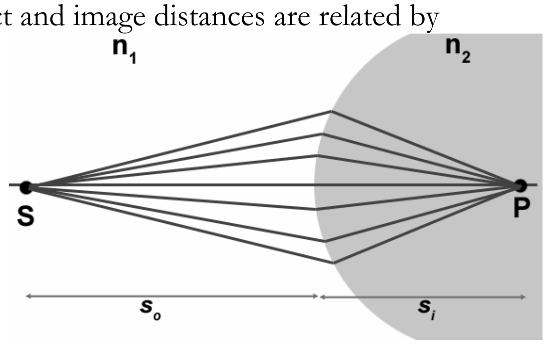
University of Oregon; Winter 2008

NAME: *Solutions -- RP*

Physics 352 – Part 1 – Optics

MIDTERM EXAMINATION

Format: Closed book. No calculators. There are 4 problems.
Total Time: 50 minutes.
Write clearly!

<table>
<tr>
<td>N-slit interference:
$$I(\theta) = I_0 \frac{\sin^2\left(N\pi D\sin(\theta)/\lambda\right)}{\sin^2\left(\pi D\sin(\theta)/\lambda\right)}$$</td>
<td>Single-slit interference:
$$I(\theta) = I_0\left(\frac{\sin(\beta)}{\beta}\right)^2,$$ where
$\beta = \pi a \sin\theta/\lambda$.</td>
</tr>
<tr>
<td>For a convex spherical interface of radius R, the object and image distances are related by


$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{(n_2 - n_1)}{R}$$
in the paraxial regime. The distances s_o and s_i are measured as positive to the left and right of the interface, respectively.</td>
<td>The thin lens formula for a lens of index of refraction n_{lens} in air:
$$\frac{1}{s_o} + \frac{1}{s_i} = (n_{lens} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right),$$
where R_1 and R_2 are the radii of curvature of the left and right lens surfaces, respectively.</td>
</tr>
</table>

Some trigonometric identities:

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v; \quad \cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

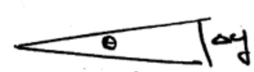
$$1 + \cos 2x = 2\cos^2 x$$

(1, 3 pts.**) Spy Satellites.** You're in charge of purchasing technology for a spy organization. In a back alley, Mr. K comes to you with a photo of two oryxes, shown below. (An oryx is a species of antelope.) He claims the photo was taken by a satellite, specifically one with a telescope mirror 1 m in diameter, orbiting 20,000 km above the Earth's surface, observing visible light (wavelength $\lambda \approx 0.5\times10^{-6}$ m). Do you believe Mr. K? Why or why not? Be quantitative, but note that a **very rough** calculation is sufficient. You do not need a calculator.

Figure 1. Two oryxes. From Google Earth (24 57' 17.99"S, 15 51' 30.54"E, in Namibia).

We can resolve the horns in the photo.

Their size $\Delta y \approx 0.1$ m.



The corresponding angular width $\theta_h \approx \frac{\Delta y}{L} = \frac{10^{-1}\,m}{2\times10^{7}\,m}$

$= \frac{1}{2}\cdot 10^{-8}$ radians

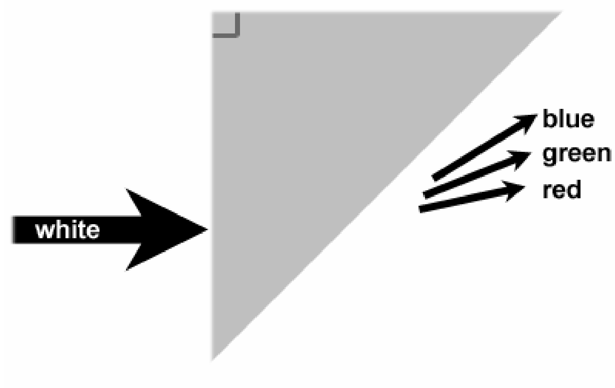
The satellite's angular resolution $\theta_r \approx \frac{\lambda}{a} = \frac{\frac{1}{2}\cdot10^{-6}\,m}{1\,m}$

$= \frac{1}{2}\cdot 10^{-6}$ radians

$\theta_r \gg \theta_h$, so there's no way we should see a sharp image of the horns.

Don't believe Mr. K.

(2, 6 pts.**) A Prism.** A quartz prism with the shape of a right isosceles triangle – the corners have angles of 90°, 45°, and 45° – is surrounded by vacuum. We shine white light normally (i.e. perpendicularly) incident on one of the legs. See the figure.

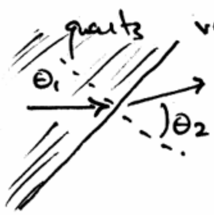


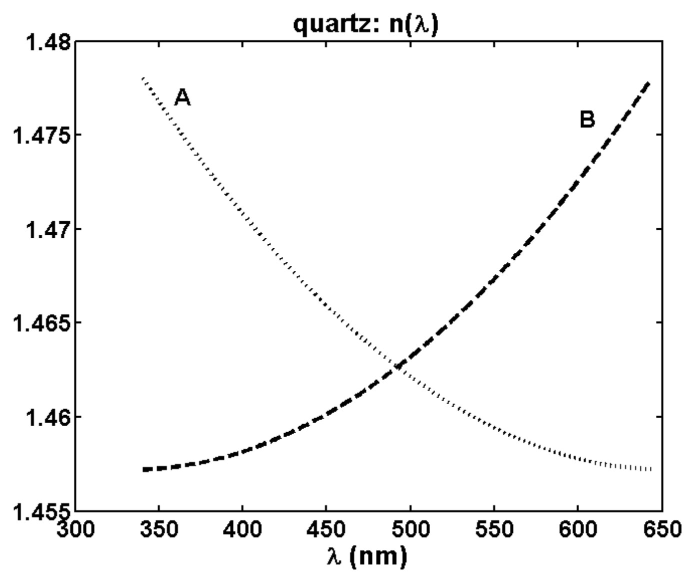
As you probably learned long ago, white light is composed of light of many colors. We see **colored rays exit the prism as indicated in the figure**, i.e. with blue ($\lambda \approx 450$ nm) "on top" and red ($\lambda \approx 650$ nm) "below."

Quartz, like all materials, has slightly different indexes of refraction for different wavelengths of light; in other words, n **is a function of** λ**.** Below I've shown two possible graphs of $n(\lambda)$ for quartz – a dotted plot (A) and a dashed plot (B). One is correct; the other is wrong. Based on the arrangement of the colored rays noted above, **which curve is correct? Explain.** (As usual, random guesses won't earn any points.) *Comment:* Despite the presence of numbers above, this problem does not require quantitative calculations; think about the shape of $n(\lambda)$.

Normal incidence → no refraction at the first interface

At the second:





[redacted]

Snell:

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

↑quartz ↑vac. $n=1$

$\theta_1 = 45°$

→ $\sin\theta_2 = n_{quartz} \sin\theta_1$

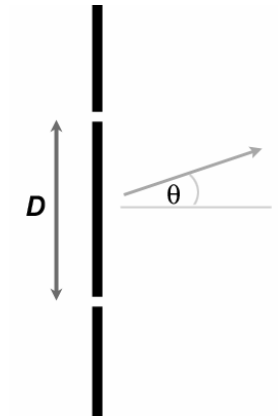
* So larger n_{quartz} → larger $\sin\theta_2$ → larger θ_2

We see that blue → larger angles,

so blue (small λ) must have higher n than red.

⇒ plot A is correct.

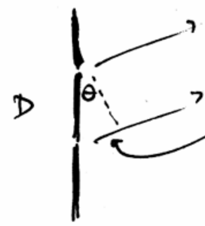
(3, 4 pts.**) 3-slit interference.** Consider light diffracted by a barrier with two slits of negligible width. The setup is the usual one: a plane wave of wavelength λ is incident from the left; the separation between the slits is D; we're concerned with the intensity hitting a far-off screen to the right. (See the figure.) Also as usual, consider small θ.



A detector is placed on the screen at the smallest positive angular position θ for which the intensity is <u>zero</u>. Call this angle θ^*.

Mr. K carves a new slit halfway between the two existing slits. (The new slit is also of negligible width.) The detector position is unchanged. Will the intensity measured by the detector at θ^* be zero, maximal, or something in between? (*Note:* if it's something in between, you **don't** have to calculate what than "something" is.)

Intensity minimum → $\frac{1}{2}\lambda$ path length difference



path length $D\sin\theta^* = \frac{\lambda}{2}$

$\rightarrow \theta^* \approx \frac{\lambda}{2D}$.

New slit.

Two approaches (either is fine):

(i) Slit "#3" ~~[illegible]~~ creates a path length difference $\frac{D}{2}\sin\theta^*$ with respect to #1 & #2

$\frac{D}{2}\sin\theta^* = \frac{\lambda}{4}$ — neither an integer nor half-integer multiple of λ

$\Rightarrow$ neither max. nor zero intensity @ θ^*

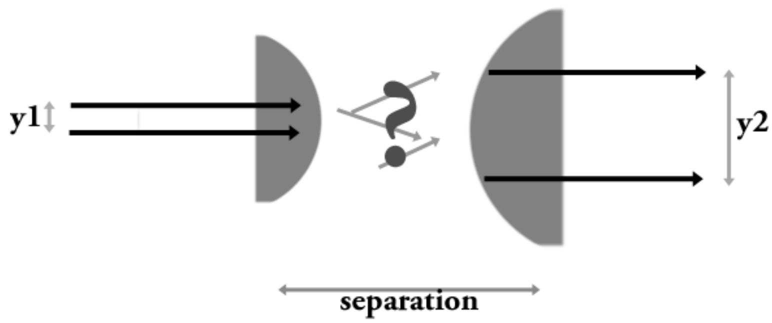
(ii) 3-slit $I(\theta) = I_0 \dfrac{\sin^2\left(3\pi\left(\frac{D}{2}\right)\sin\theta/\lambda\right)}{\sin^2\left(\pi\left(\frac{D}{2}\right)\sin\theta/\lambda\right)}$ (slit separation $D/2$)

$\theta = \theta^*$: $I(\theta^*) = I_0 \dfrac{\sin^2(3\pi/4)}{\sin^2(\pi/4)} = I_0 \dfrac{(1)}{(1)}$.

<u>Not</u> a zero, <u>nor</u> a max $\left(\frac{0}{0}\right)$.

$\Rightarrow$ "in between" intensity.

(4, 8 pts.**) Keplerian Telescope.** Consider the pair of thin glass ($n = 1.5$) lenses in air, as shown below. This arrangement is known as a Keplerian Telescope. The first, a <u>planar-convex</u> lens, has a radius of curvature $|R_A| = 20$ cm on the curved side. The second, <u>also a planar-convex lens</u>, has a radius of curvature $|R_B| = 80$ cm on the curved side.



Suppose a plane wave is incident from the left, as indicated by the rays drawn, and we want a plane wave to exit from the right, as drawn.

(a, 6 pts.**)** What separation, d, must the lenses have? Express your answer (*i*) symbolically in terms of n, R_A, and R_B, and (*ii*) numerically given the above parameters. *Suggestion:* Consider each lens independently, and see also part (b).

Thin lens: $\frac{1}{s_o} + \frac{1}{s_i} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.

Lens 1: $R_1 = \infty$, $R_2 = R_A = -20$ cm. $s_o = \infty$

$\rightarrow \frac{1}{\infty} + \frac{1}{s_i} = (n-1)\left(\frac{1}{-R_A}\right) \rightarrow s_i = -\frac{R_A}{n-1} = \frac{+20\text{ cm}}{0.5} = +40$ cm.

Lens 2: $R_1 = +80$ cm $= R_B$, $R_2 = \infty$. $s_i = \infty$. $s_o = d - s_{o,1}$

$\Rightarrow \frac{1}{d - s_{o,1}} + \frac{1}{\infty} = (n-1)\left(\frac{1}{R_B}\right) \rightarrow d - s_{o,1} = \frac{R_B}{n-1}$

$\rightarrow d = s_{o,1} + \frac{R_B}{n-1} = +40\text{ cm} + \frac{+80\text{ cm}}{0.5} = 200$ cm.

$\boxed{d = \frac{(-R_A + R_B)}{(n-1)}}$; $\boxed{d = 200\text{ cm}}$

(b, 2 pts.**)** Draw a diagram of the lenses that includes the rays in between the lenses, and indicate any real or virtual image points.